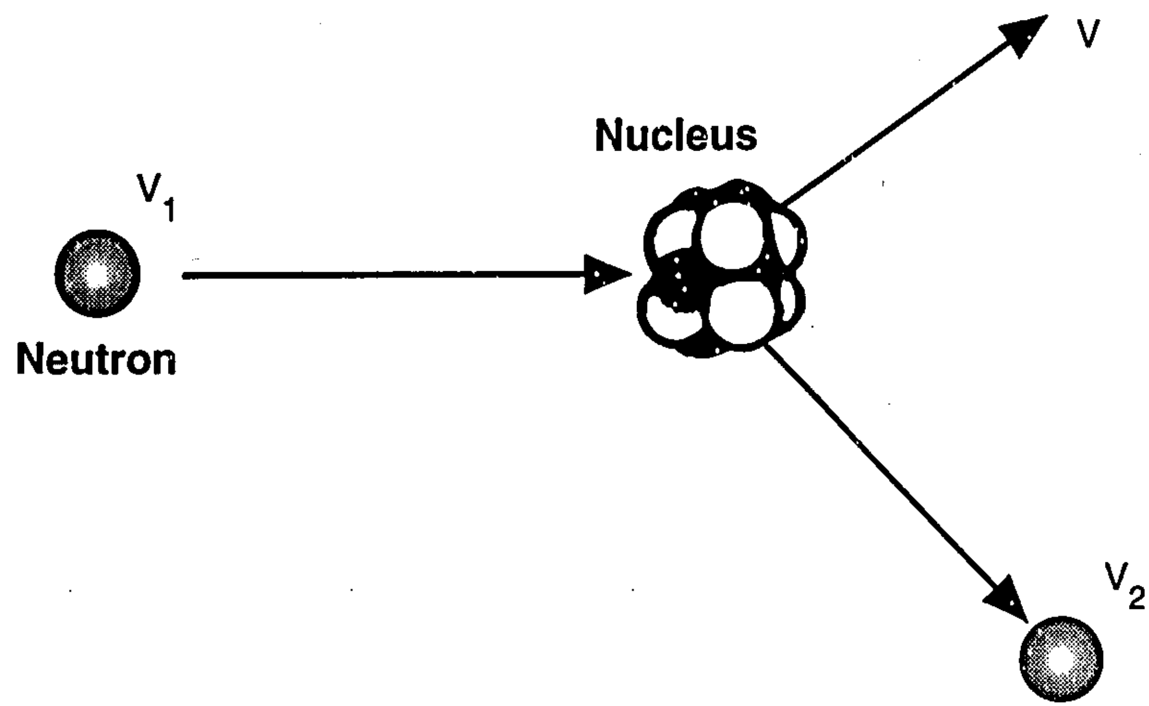
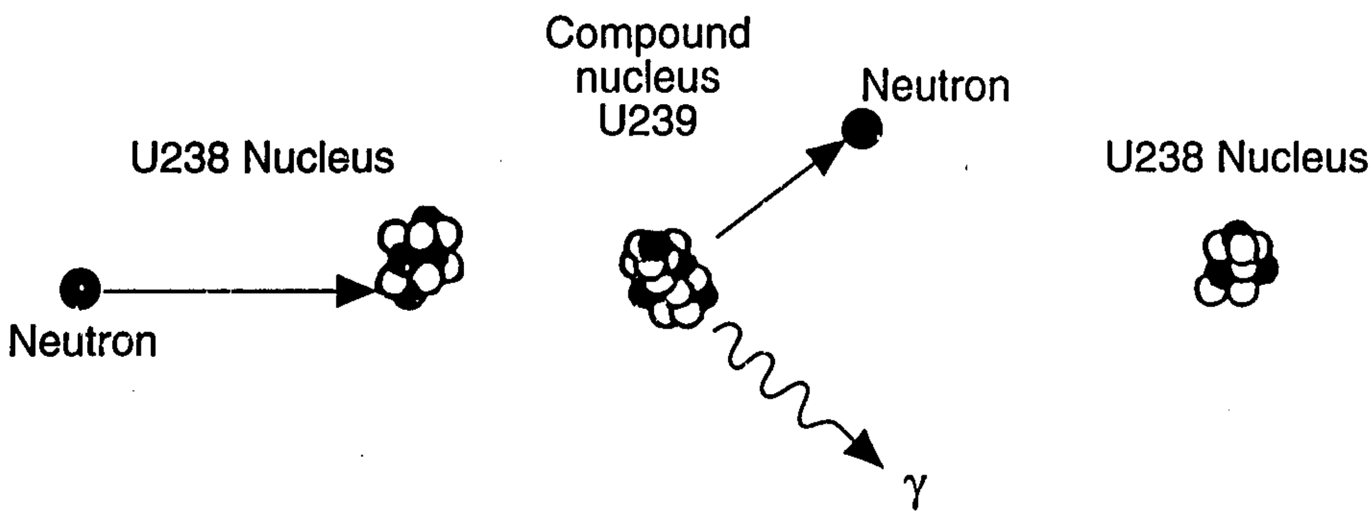
Elastic Collison



Inelastic Scattering



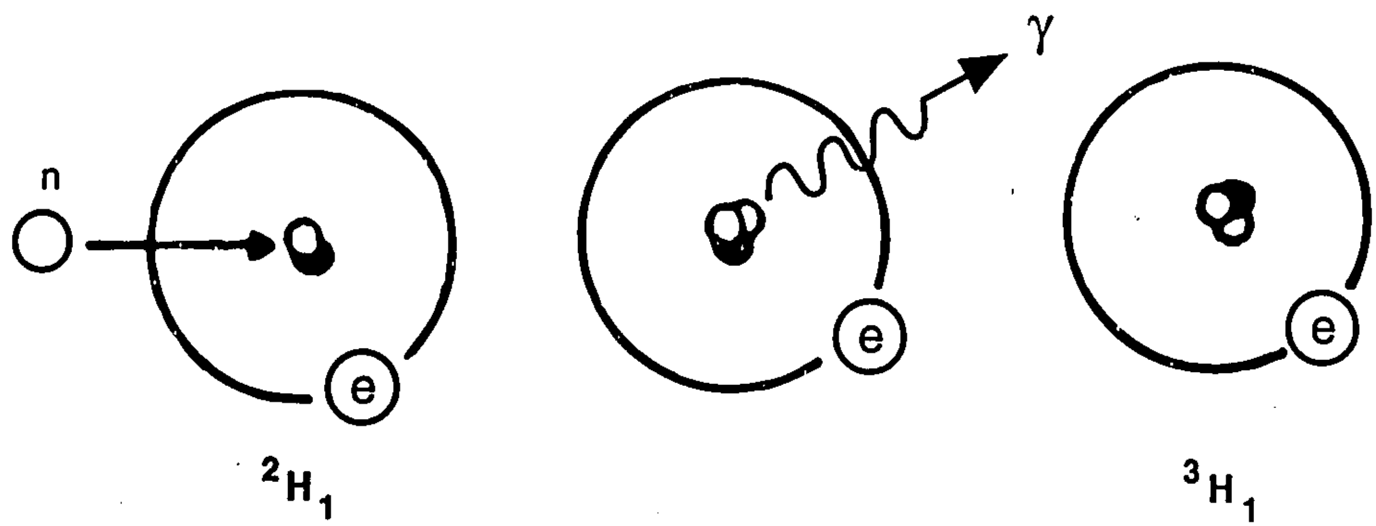
Transmutation

^{1}Neutron$_{0}$ + $^{16}0_{8}$Nucleus $^{16}N_{7}$Nucleus + ^{1}Proton$_{1}$

This reaction may be written as

$$^{1}n_{0} + {}^{16}0_{8} \longrightarrow {}^{16}N_{7} + {}^{1}p_{1}$$

Radiative Capture



Fission Yield of U-235 and Pu-239

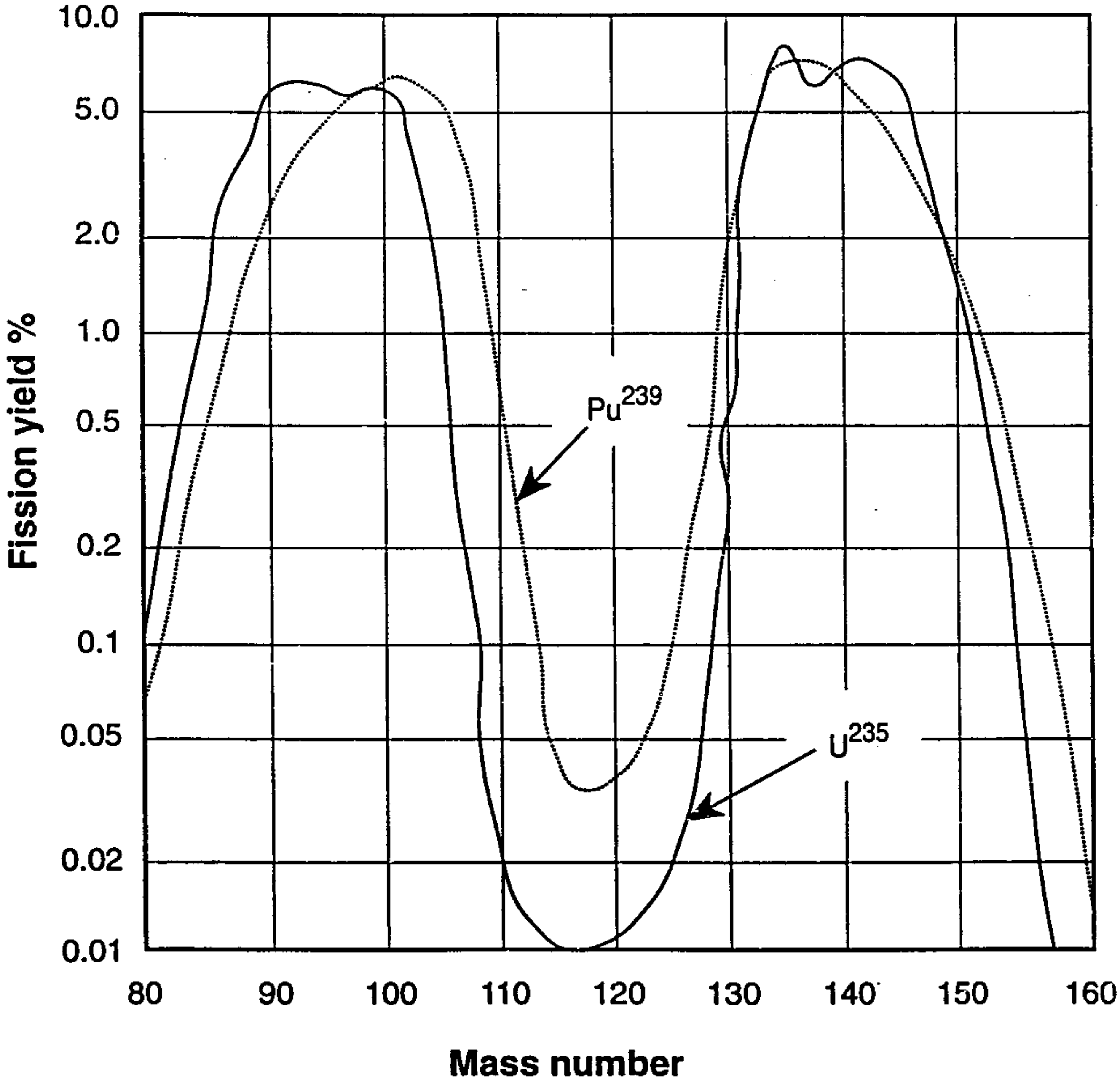


Fig. :

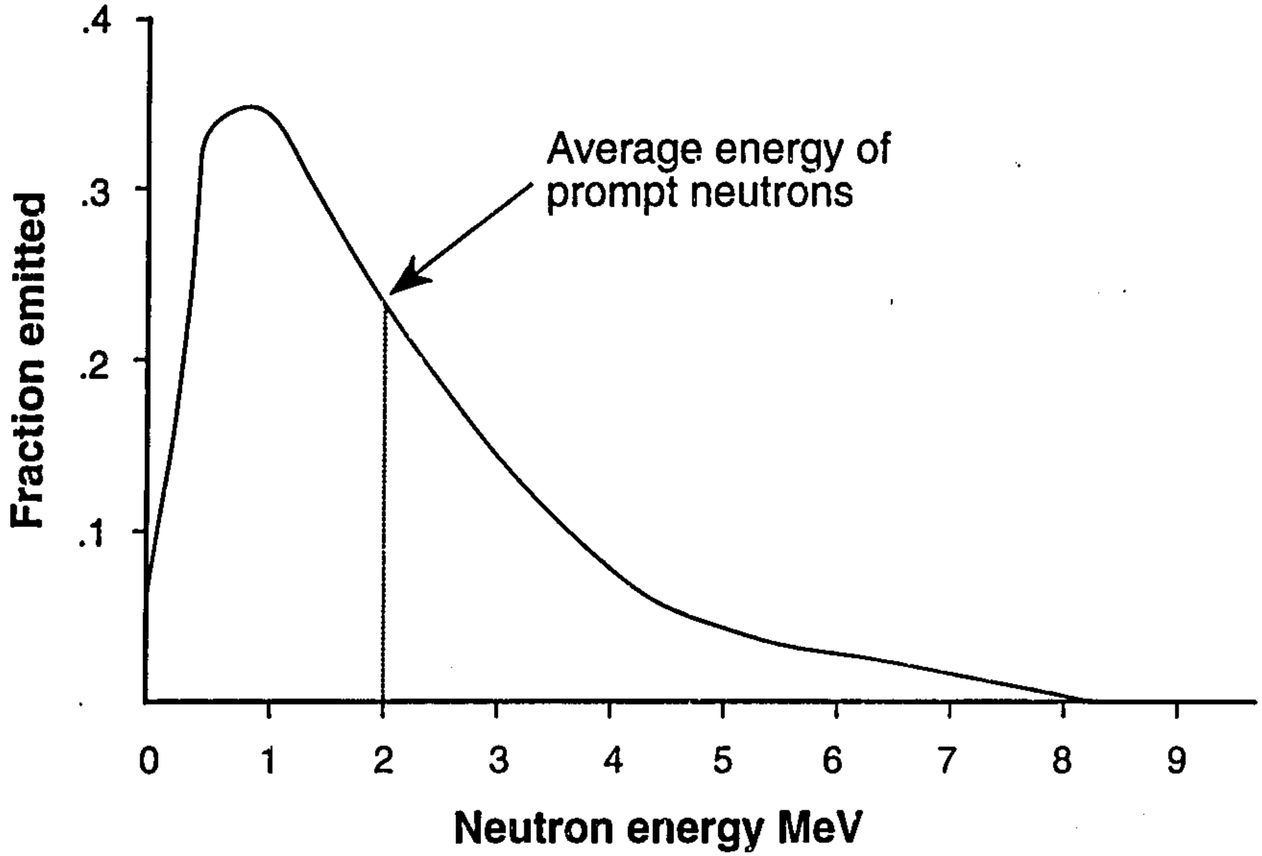
Fission Product Decay Chain

$$n + {}^{235}U_{92} \longrightarrow {}^{139}Xe_{54} \xrightarrow[\gamma]{\beta^-} {}^{139}Cs_{55} \xrightarrow[\gamma]{\beta^-} {}^{139}Ba_{56} \xrightarrow[\gamma]{\beta^-} {}^{139}La_{57}$$

$$n + {}^{235}U_{92} \longrightarrow n + n + \gamma$$

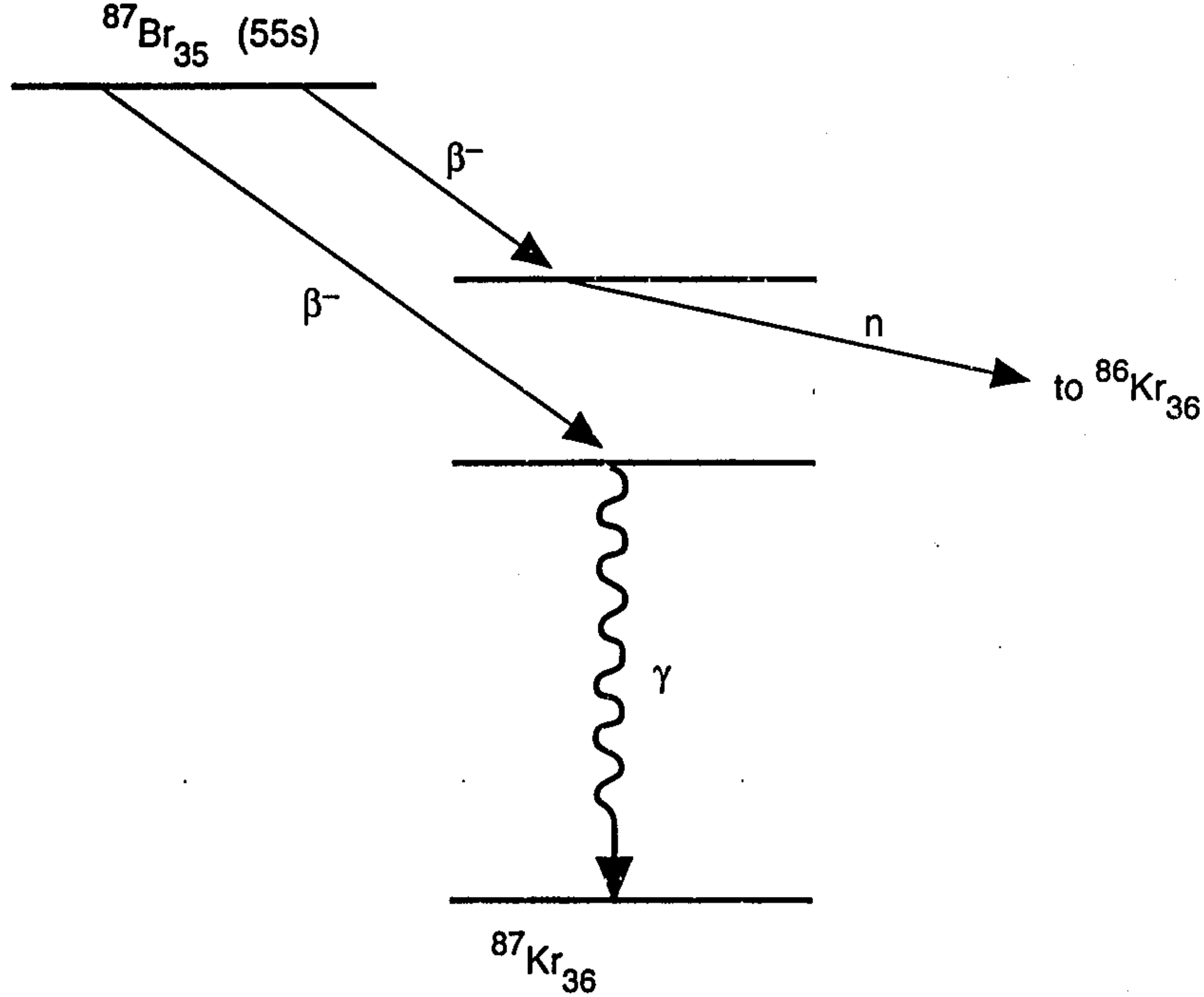
$$n + {}^{235}U_{92} \longrightarrow {}^{95}Sr_{38} \xrightarrow[\gamma]{\beta^-} {}^{95}Y_{39} \xrightarrow[\gamma]{\beta^-} {}^{95}Zr_{40} \xrightarrow[\gamma]{\beta^-} {}^{95}Nb_{41} \xrightarrow[\gamma]{\beta^-} {}^{95}Mo_{42}$$

Energy Distribution of Prompt Fission Neutrons

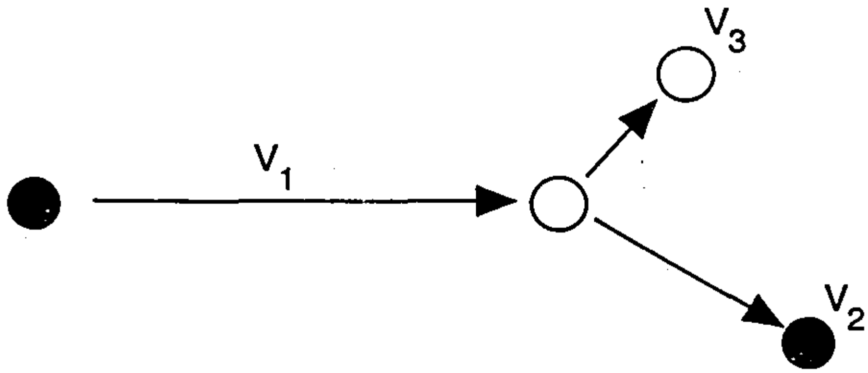


Production of Delayed Neutron From Br-87

(normal beta-gamma decay mode also shown)

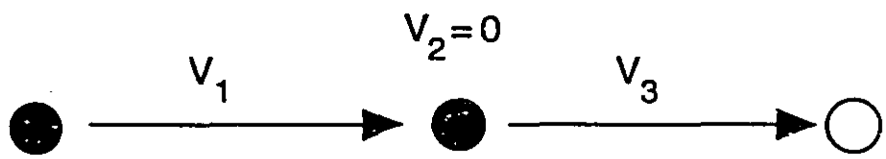


Elastic Collisions
(for equal masses)

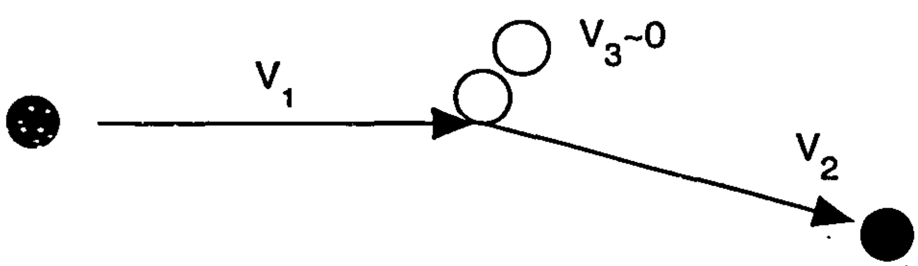


$$1/2m\ V_1^2 = 1/2m\ V_2^2 + 1/2m\ V_3^2$$

(In all three cases)



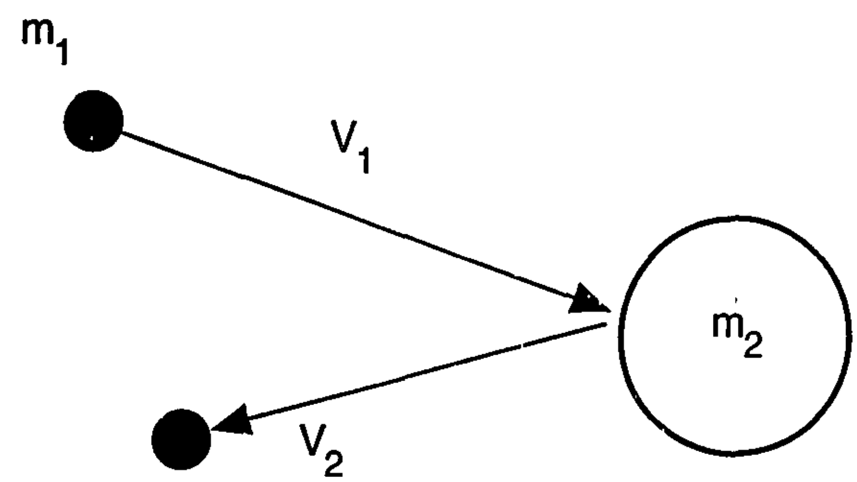
Maximum transfer of energy
(head-on collision)



Very small transfer of energy
(glancing collision)

Elastic Collision

$(m_2 >> m_1)$



If $m_2 >> m_1$,

$V_1 \sim V_2$

and

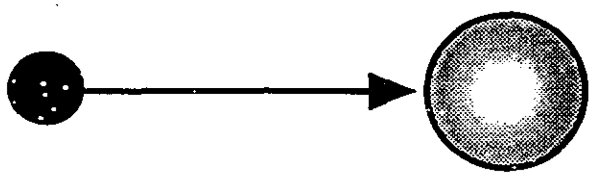
$V_3 \sim 0$

Average Number of Elastic Collisions to Thermalize Fission Neutrons in Various Materials

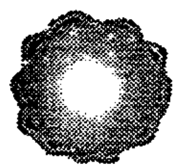
H	18
D (Deuterium)	25
H_20 (Light water)	20
D_20 (Heavy water)	36
C-12 (Graphite)	115
U-238	2172

Inelastic Scattering
(internal reaction)

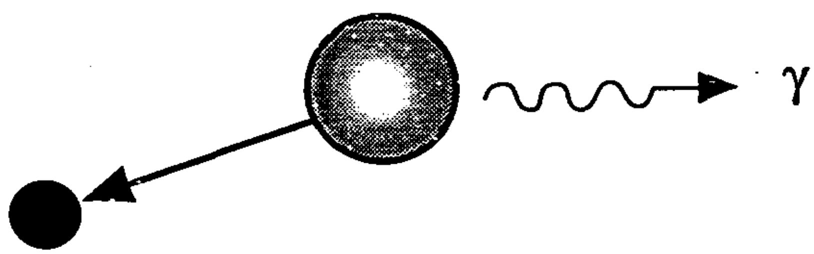
1. Neutron enters stable nucleus



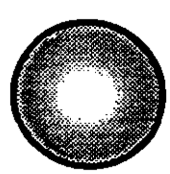
2. Nucleus is excited



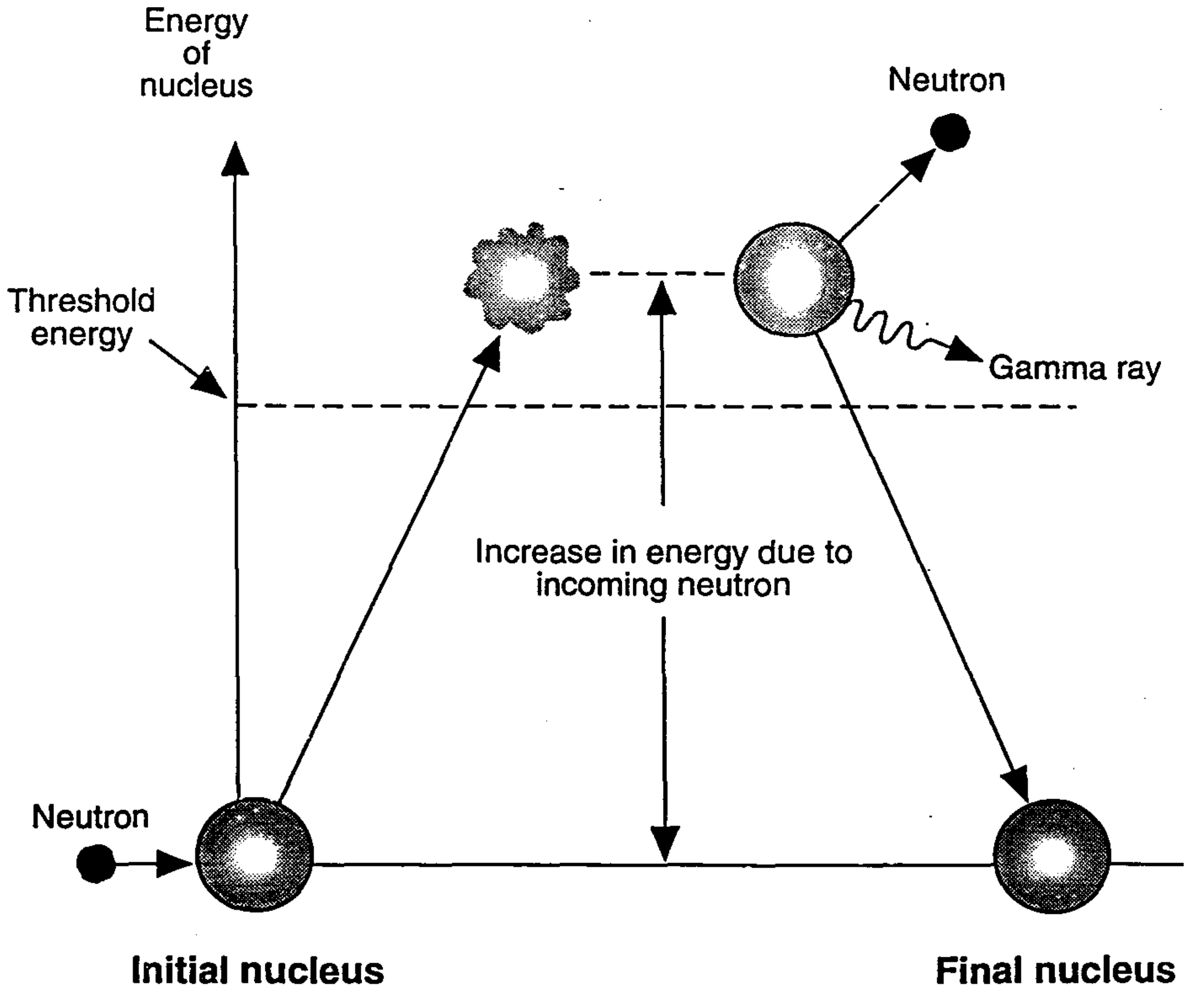
3. Neutron and γ - photon are emitted



4. Nucleus is stable again

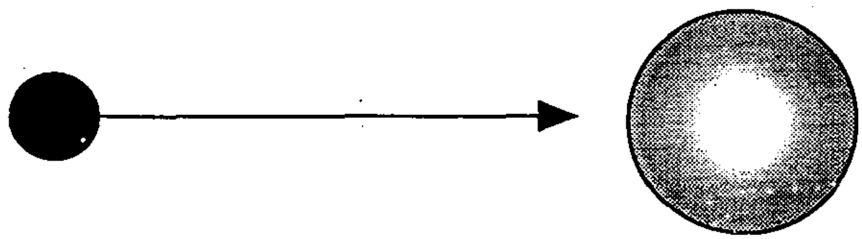


Inelastic Scattering
(above threshold energy)



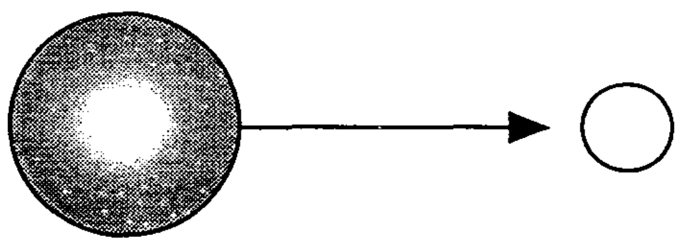
Nuclear Transmutation

1. Neutron enters stable nucleus



$^{1}\text{Neutron}_{0} + {}^{16}\text{O}_{8}$ Nucleus $\longrightarrow$ $^{17}\text{O}_{8}$ Nucleus

2. Another particle leaves excited nucleus



$^{17}\text{O}_{8}$ Nucleus $\longrightarrow$ $^{16}\text{N}_{7}$ Nucleus + $^{1}\text{Proton}_{1}$

This may be written as

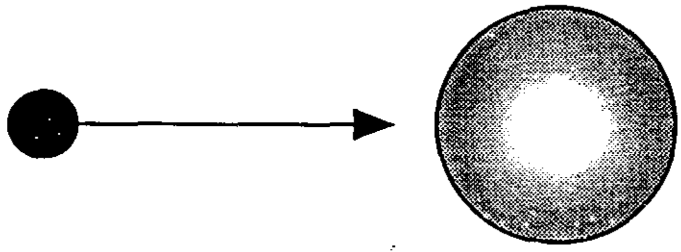
$$^{1}n_{0} + {}^{16}O_{8} \longrightarrow {}^{16}N_{7} + {}^{1}p_{1}$$

$$^{16}O\ (n,p)\ ^{16}N$$

(^{16}N nucleus subsequently emits gamma rays)

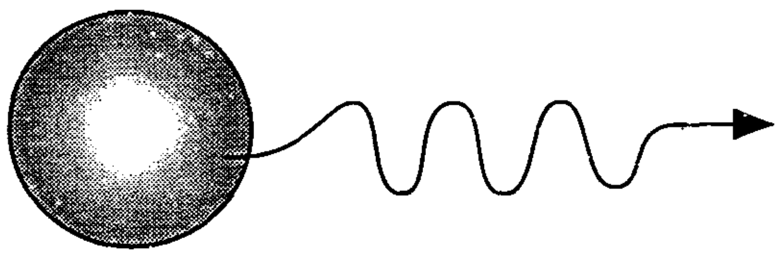
Radiative Capture

1. Neutron enters stable nucleus



$$^{1}n_{0} + {}^{238}u_{92} \longrightarrow {}^{239}u^{*}_{92}$$

2. Gamma ray leaves excited nucleus



$$^{239}u^{*}_{92} \longrightarrow {}^{239}u_{92} + \gamma$$

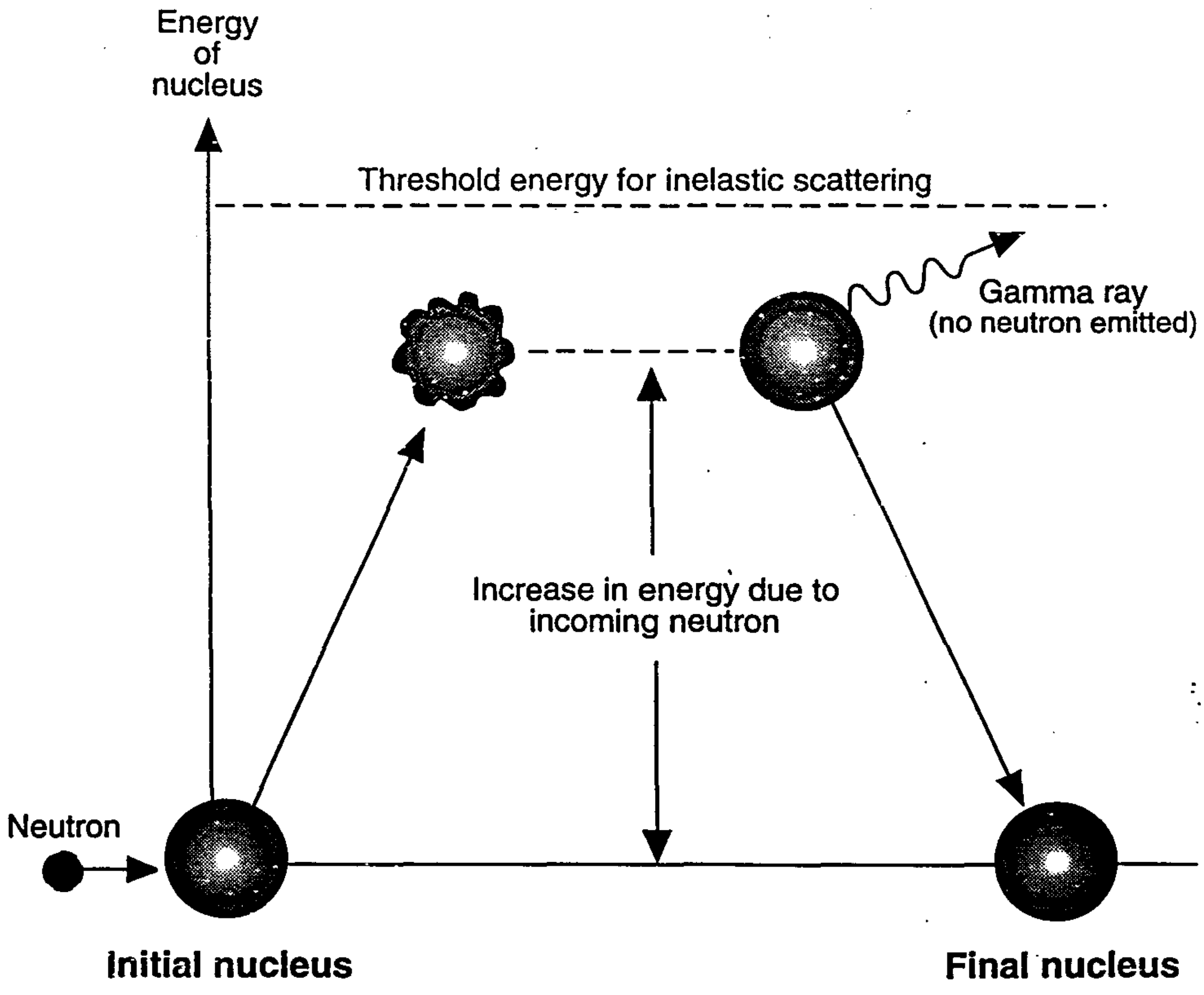
This may be written as

$$^{1}n_{0} + {}^{A}X_{Z} \longrightarrow {}^{A+1}X_{Z} + \gamma$$

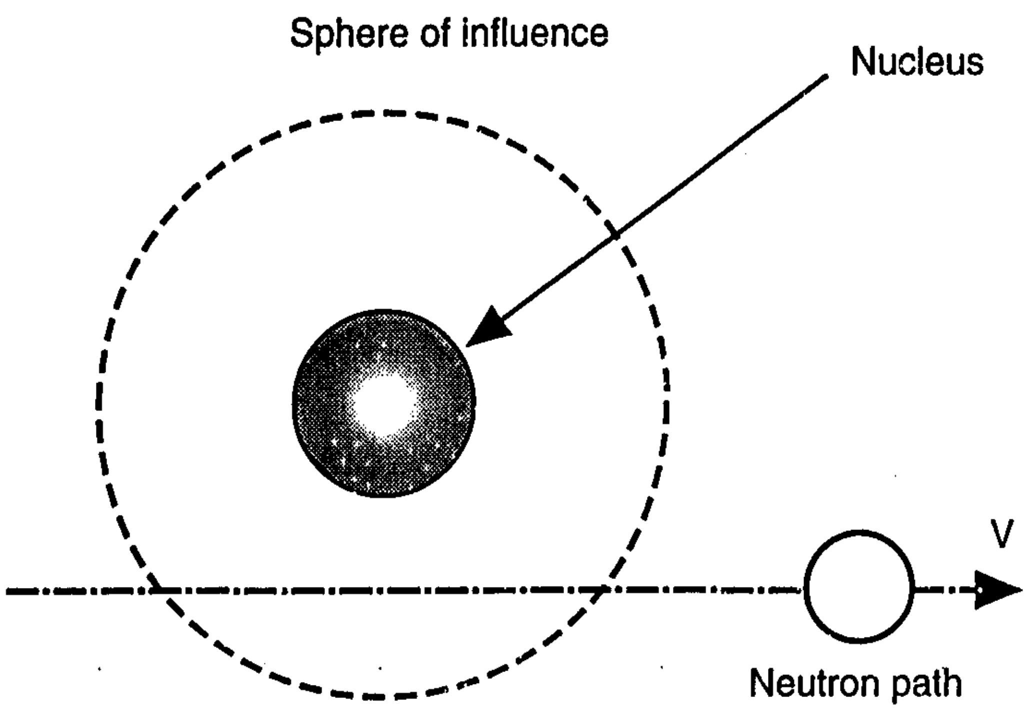
$$^{238}u\ (n,\gamma)\ {}^{239}u$$

$^{239}u_{92}$ subsequently transmutes to $^{239}Pu_{94}$. How?

Radiative Capture
(below threshold energy)



Radiative Capture



Time spent in sphere of influence

$$t \propto \frac{1}{v}$$

If probability of capture $\propto$ time

$$\sigma_a \propto \frac{1}{v}$$

Spontaneous Fission and Alpha Decay Rates of Uranium

	$t_{1/2}$ (α) (years)	$t_{1/2}$ (s.f.) (years)	α decay rate (atoms /s/kg)	s.f. decay rate (atoms /s/kg)
U-235	7.1×10^8	1.2×10^{17}	79×10^6	0.3
U-238	4.5×10^9	5.5×10^{15}	12×10^6	6.9

Fissile Materials

Fuel	Neutrons per fission, υ	Neutrons per absorption, η
Uranium-233	2.51	2.28
Uranium-235	2.43	2.07
Natural uranium	2.43	1.34
Plutonium-239	2.90	2.10
Plutonium-241	3.06	2.24

Formation of Plutonium

$$^{238}U_{92} + n \longrightarrow {}^{239}U_{92} + \gamma$$

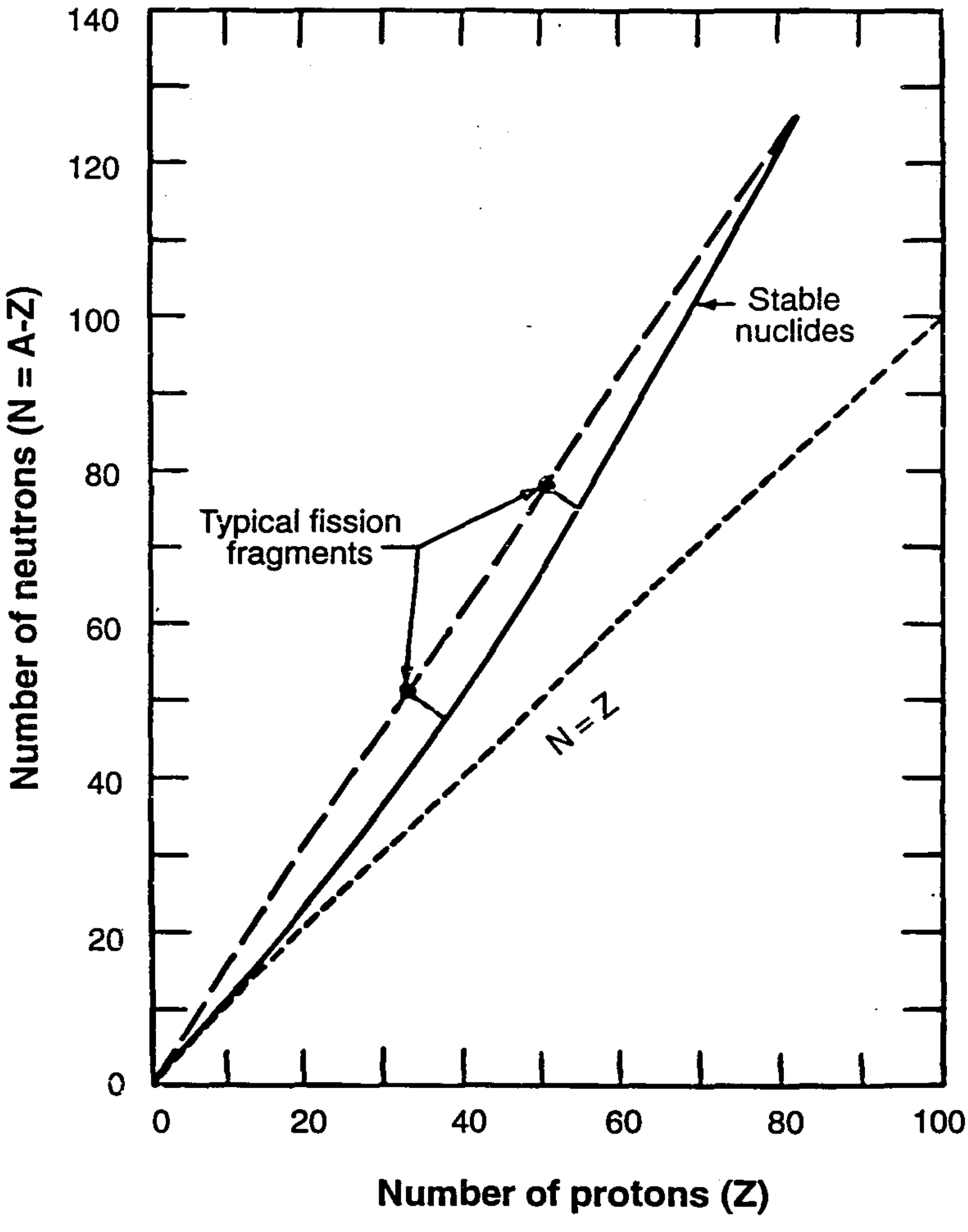
$$\xrightarrow{t_{1/2} = 24m}$$

$$^{239}Np_{93} + \beta^-$$

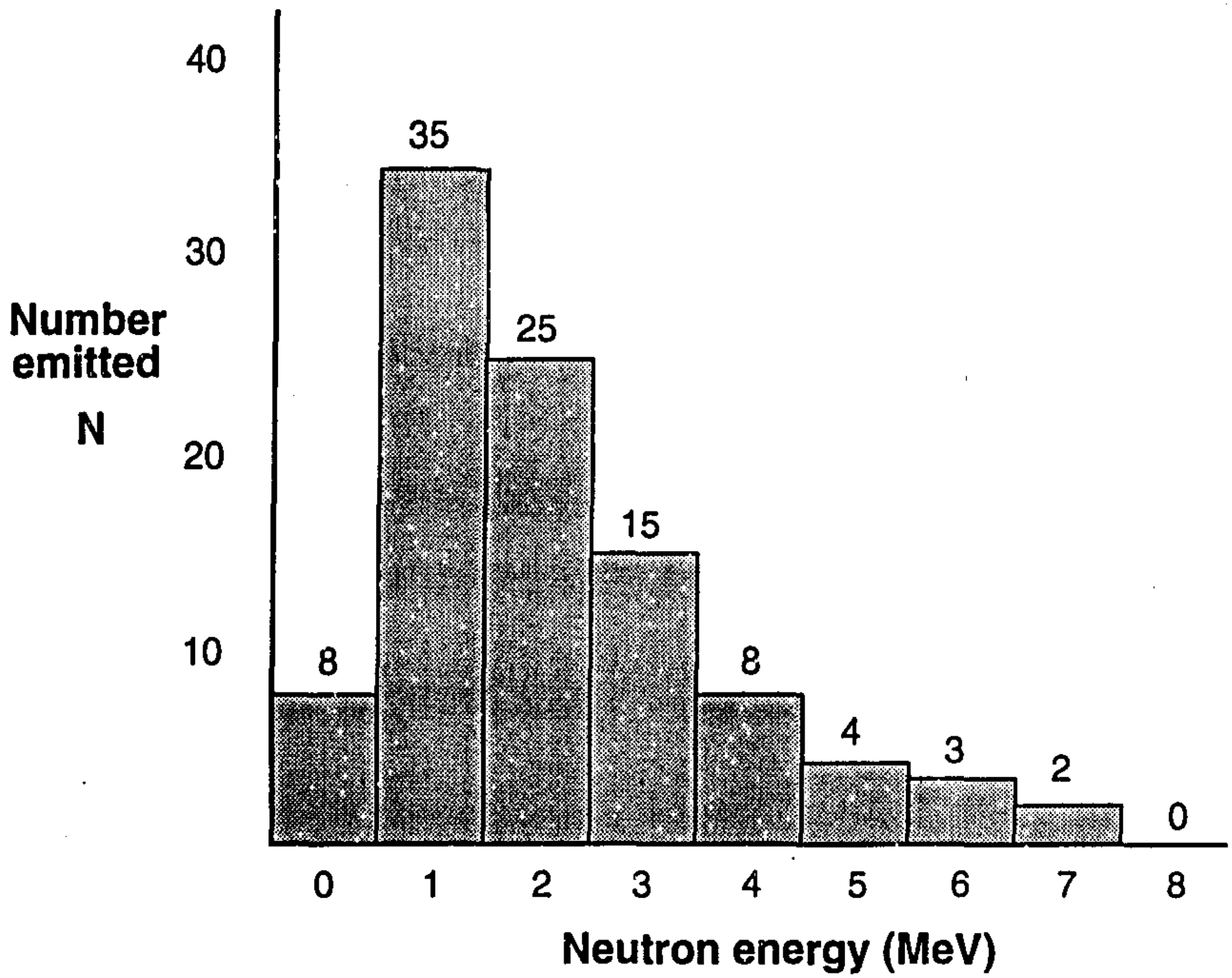
$$\xrightarrow{t_{1/2} = 2.4d}$$

$$^{239}Pu_{94} + \beta^-$$

Reason for Instability of Fission Fragments



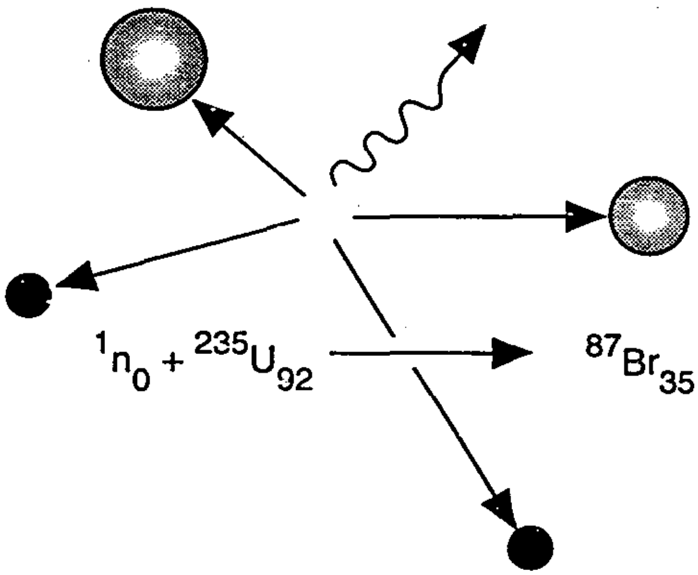
Energy Distribution
Sample of 100 Prompt Neutrons



Delayed Neutrons

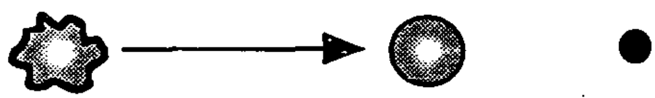
Delayed neutrons come from certain nuclei formed by beta decay of fission products

Fission:



$$^{1}n_{0} + {}^{235}U_{92} \longrightarrow {}^{87}Br_{35} + {}^{147}La_{57} + {}^{1}n_{0} + {}^{1}n_{0} + \gamma$$

Decay:



$$^{87}Br_{35} \longrightarrow {}^{87}Kr_{36} + {}^{0}e_{-1}$$

$$^{87}Kr_{36} \longrightarrow {}^{86}Kr_{36} + {}^{1}n_{0}$$

Delayed Neutron Precursor Groups for U-235 Fission

Precursor group	Half-life (seconds)	Relative yield (%)
1	55	4
2	22	21
3	6	19
4	2.2	40
5	0.5	13
6	0.18	3

Secondary Neutron Emission in Thermal Fission of U-235

Number of neutrons emitted	Number of cases per 1000 fissions
0	27
1	158
2	339
3	302
4	130
5	34

Value of υ for Thermal Fissions

Fissile nucleus	υ
U-235	2.43
Pu-239	2.89
Pu-241	2.93

Approximate Distribution of Fission Energy Release in U-235

Kinetic energy of lighter fission fragment	100 MeV
Kinetic energy of heavier fission fragment	69 MeV
Kinetic energy of secondary neutrons	5 MeV
Energy of prompt γ rays	6 MeV
Beta particle energy gradually released from fission products	8 MeV
Gamma ray energy gradually released from fission products	6 MeV
Neutrinos (energy escapes from reactor)	11 MeV
Total	205 MeV

Location of Fission Heating

Source	Location of heating
Fission fragments	Fuel pellets (fragments slow down in a distance of about 1μm)
Kinetic energy of secondary neutrons	Mostly transferred to moderator by collisions with moderator atoms
Beta particles from fission product decay	Fuel pellets and cladding
Prompt gammas and gammas from fission product decay	Throughout reactor and its shielding (about one third deposited in moderator)

Fission

Spontaneous Fission

- Rare but possible

Induced Fission

- Excited energy level must be above critical
- Adding a neutron adds energy to the nucleus (kinetic & binding energy)

Fissile Nuclei

- Fission with zero energy neutrons

Fissionable Nuclei

- Fission with energetic (fast) neutrons only

Fission Process Summary

* Critical energy of compound nucleus must be less than binding energy of added neutron

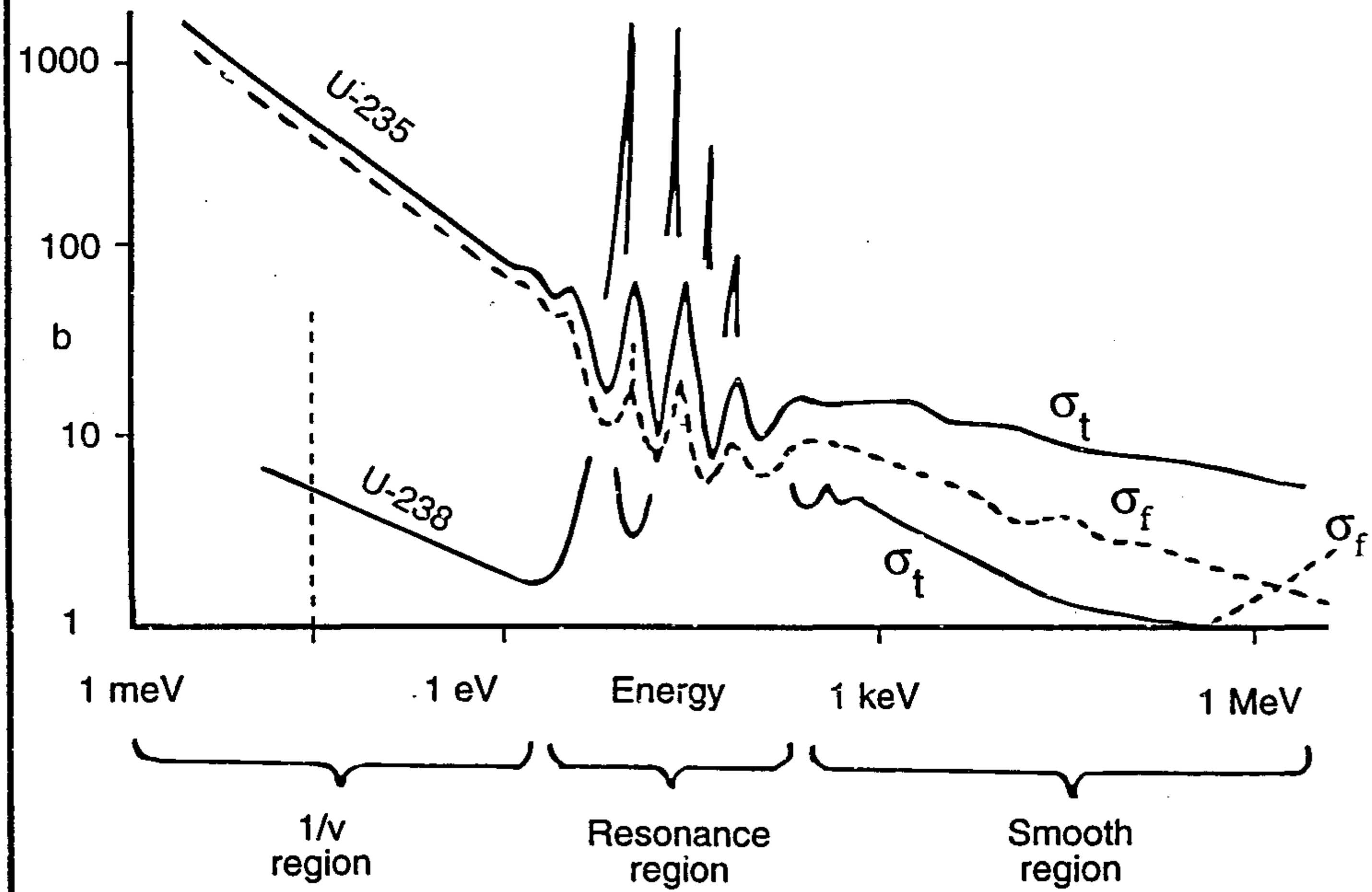
* Low energy neutrons interact more readily with U-235 to cause fission than do high energy neutrons. U-238, on the other hand, will only undergo fission with high-energy neutrons.

* Neutron/proton ratio curve results in additional neutrons being produced in fission ($\upsilon > 1$)

* Neutrons produced in fission have range of energies. $\bar{E} \approx 2\text{MeV}$.

* Neutrons must be slowed down to lower energies (thermalised or moderated) to start new cycle.

Fission Characteristics



Interactions of importance

σ_s = Scattering
σ_γ = Radiative capture
σ_f = Fission

$\left.\begin{array}{l}\sigma_s = \text{Scattering}\\ \sigma_\gamma = \text{Radiative capture}\\ \sigma_f = \text{Fission}\end{array}\right\} \sigma_a = \text{Absorption}$

Capture/fission ratio: $\alpha = \sigma_\gamma / \sigma_f$

Probability of fission: $p = \sigma_f / \sigma_a$

Avogadro's Number

$$N_A = 6.022 \times 10^{23}$$

Number of atoms or nuclei in a given sample

$$N = \frac{N_A}{A} \times \text{MASS (g)}$$

Example: Atoms in 1kg of U-235

$$N = \frac{6.022 \times 10^{23}}{235} \times 1000$$

$$= 25.62 \times 10^{23} \text{ atoms}$$

Example: 1kg of U-235 consumed in one day

$$N = \frac{6.022 \times 10^{23}}{235} \times 1000 \quad \text{fission/day}$$

$$= 25.62 \times 10^{23} \text{ atoms } / \ (24 \times 3600) \text{ Fissions/s}$$

$$= 0.0002965 \times 10^{23} \times 200 \text{ MeV/s}$$

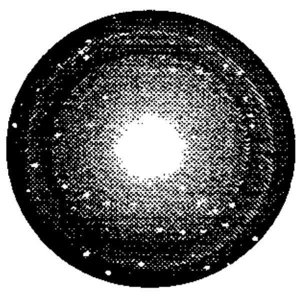
$$= 0.05932 \times 10^{23} \times 1.6022 \times 10^{-13} \text{ J/s}$$

$$= 0.09504 \times 10^{10} \text{ W}$$

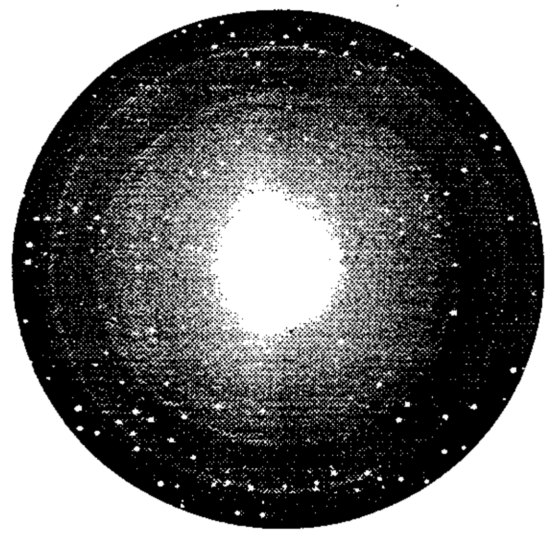
$$= 950 \times 10^{6} \text{ W}$$

$$= 950 \text{ MW}$$

Cross-Sections for U-238



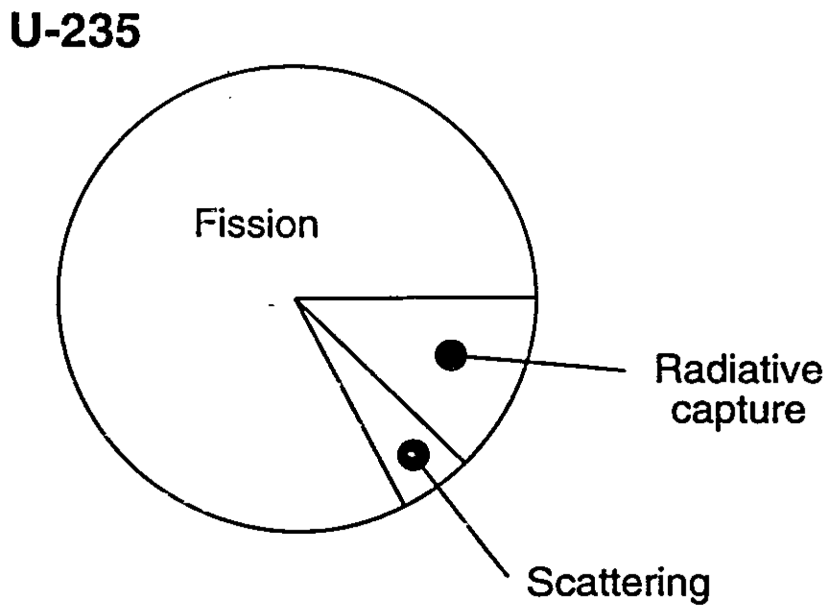
**Radiative capture
Cross-section of U-238**



**Elastic scattering
Cross-section of U-238**

Fig. 3.

Pie Diagram for Cross-Sections of U-235 and Nat–U



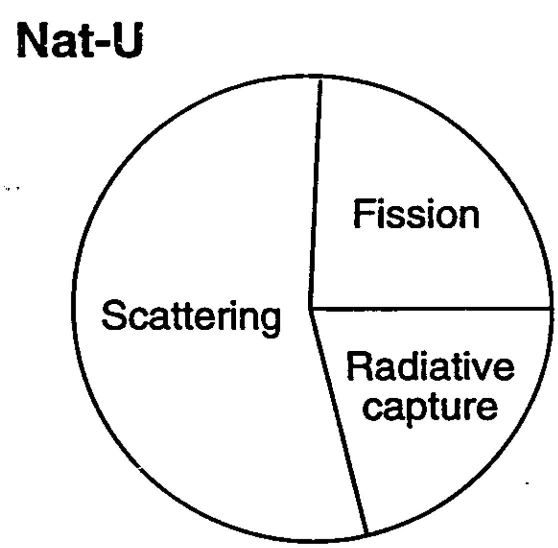
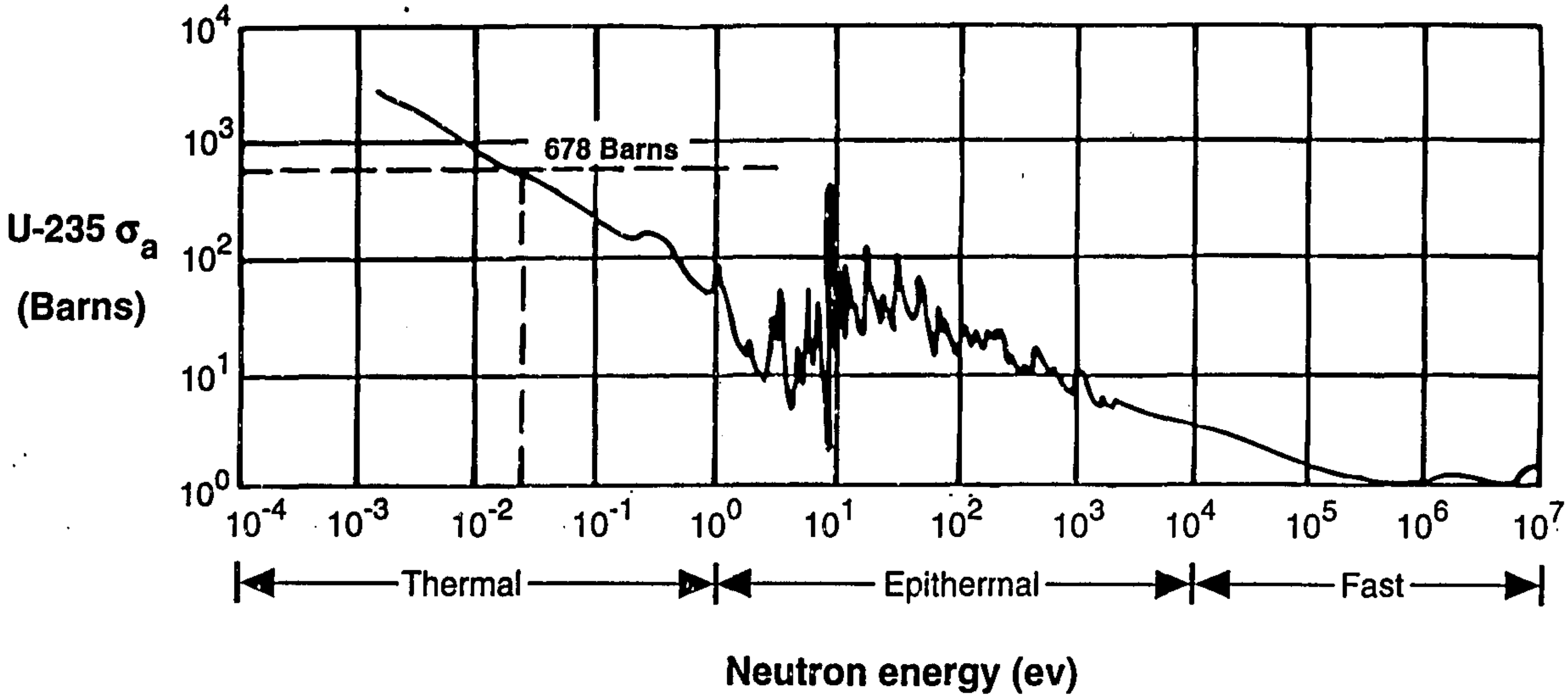
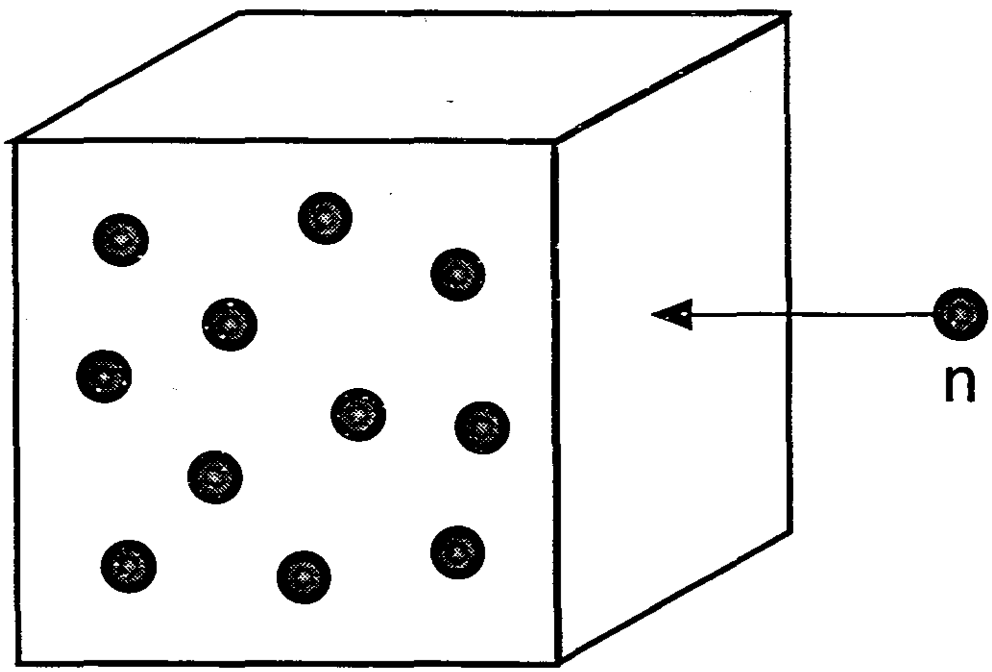


Fig. 3

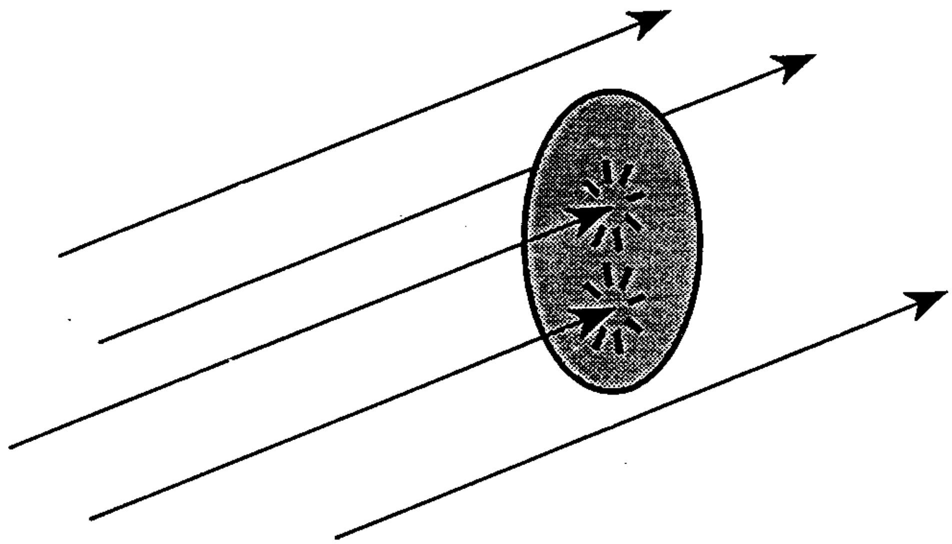
Variation of Absorption Cross-Section of U-235 with Neutron Energy



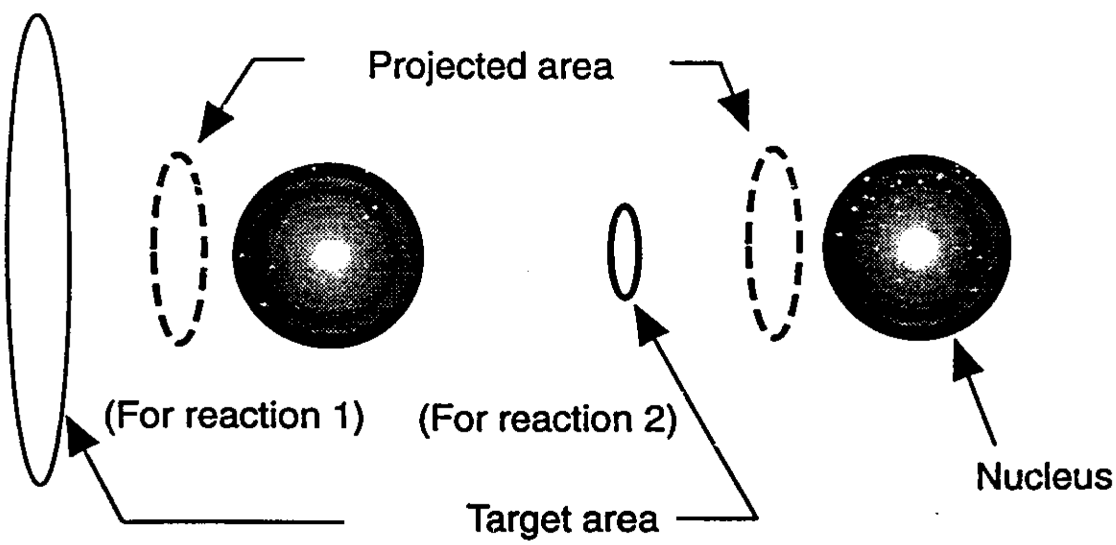
Concept of Cross-Section



Target Areas

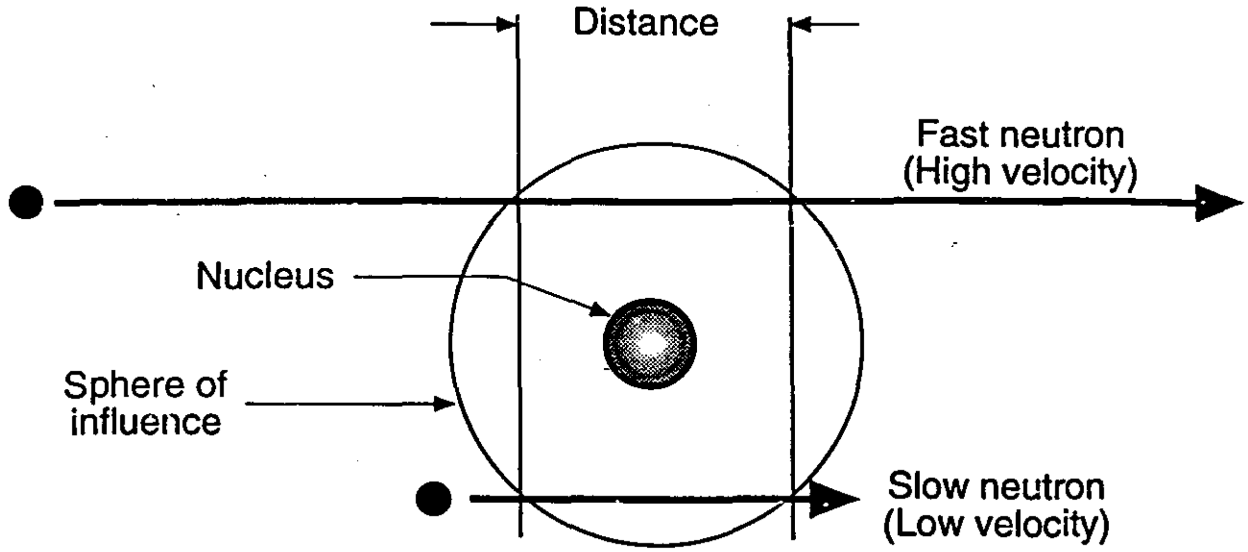


Target area may be smaller or larger than projected (physical) area of nucleus



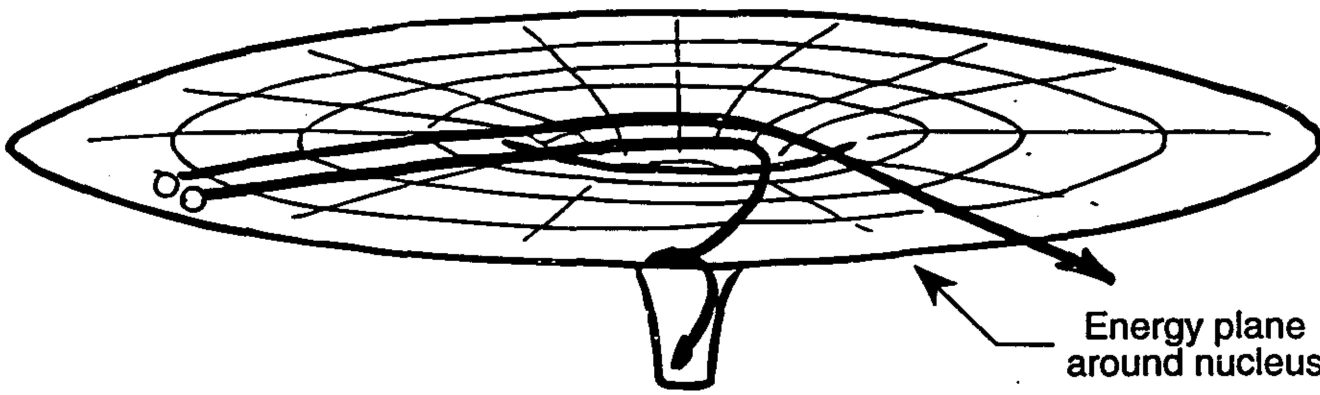
Target areas are different for different nuclear reactions

Interaction Models



Time to react with nucleus = distance/velocity

Time Model



Low velocity gives greater chance of capture

Energy Model

Comparison of Cross-Sections for Thermal Neutrons

Element	Nucleus radius $r = 1.2 \times 10^{-15} A^{1/3}$ (m)	Nucleus radius (cm)	Nucleus area $A = \pi r^2$ (cm^2)	Cross section σ_a (cm^2)	Cross section σ_s (cm^2)
H	0.0012×10^{-12}	0.12×10^{-12}	0.045×10^{-24}	0.332×10^{-24}	38×10^{-24}
B	0.0026×10^{-12}	0.26×10^{-12}	0.212×10^{-24}	759×10^{-24}	3.6×10^{-12}
C	0.0027×10^{-12}	0.27×10^{-12}	0.229×10^{-24}	0.0034×10^{-24}	4.75×10^{-24}
O	0.0030×10^{-12}	0.30×10^{-12}	0.283×10^{-24}	0.00027×10^{-24}	3.76×10^{-24}
Pb	0.0071×10^{-12}	0.71×10^{-12}	1.584×10^{-24}	0.17×10^{-24}	11.4×10^{-24}
U	0.0074×10^{-12}	0.74×10^{-12}	1.720×10^{-24}	7.53×10^{-24}	8.9×10^{-24}
U-235	0.0074×10^{-12}	0.74×10^{-12}	1.720×10^{-24}	99×10^{-24} (n,γ) 582×10^{-24} (n,f)	

Note that projected area of nucleus is about 1×10^{-24} cm = 1 Barn

Cross-Section Nomenclature

σ_f = Fission cross section

σ_a = Absorption cross section

$\sigma_{n,\gamma}$ = Radiative capture cross section

σ_i = Inelastic scattering cross section

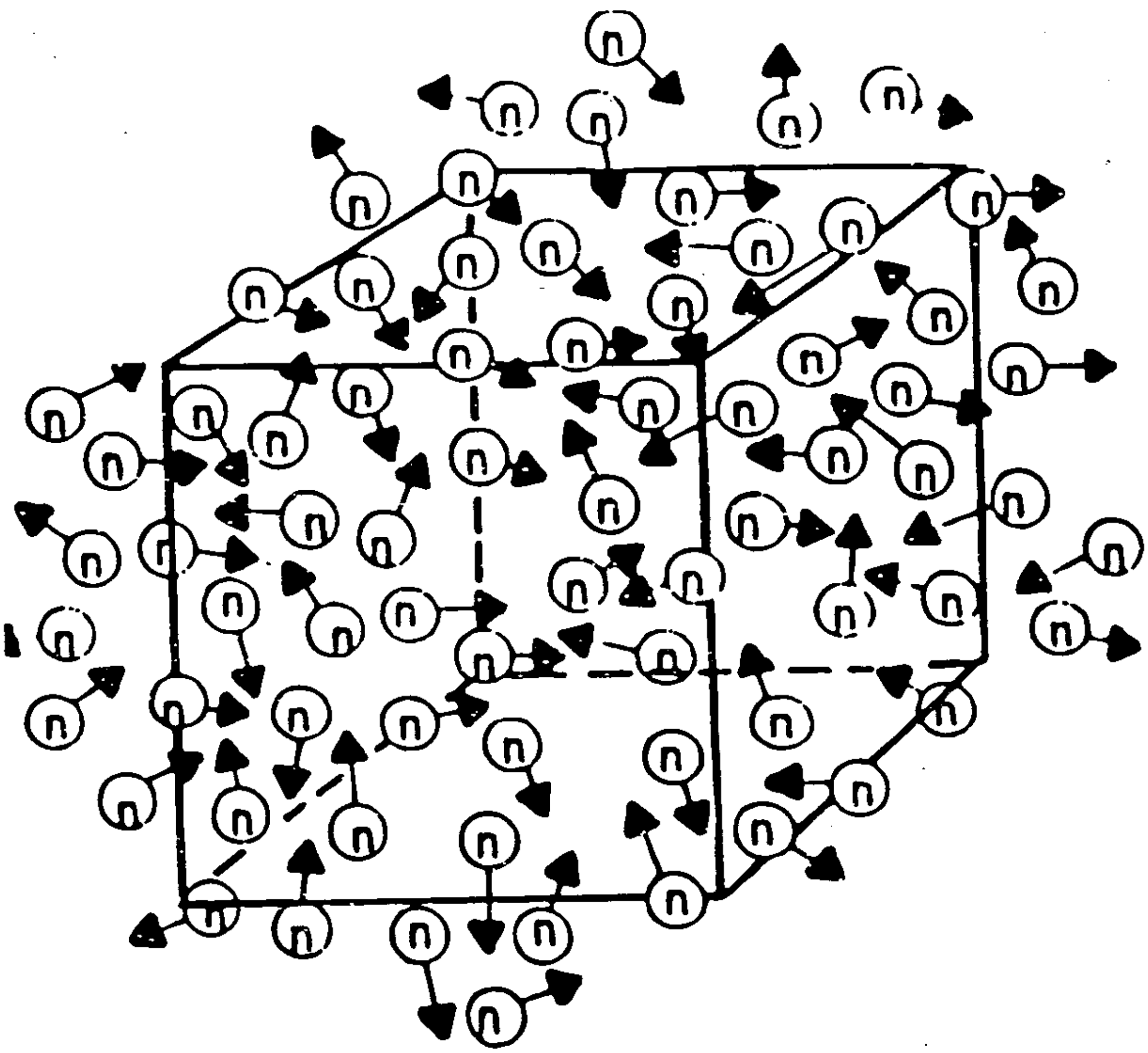
σ_s = Elastic scattering cross section

Thermal Neutron Cross-Sections of Fuel Atoms (in Barns)

Taken from Atomic Energy Review (IAEA), 1969, Vol 7, No 4, p.3

	σ_f	$\sigma_{n,\gamma}$	σ_a	σ_s	υ	σ_f/σ_a (%)
U-233	530.6	47.0	577.6	10.7	2.487	92
U-235	580.2	98.3	678.5	17.6	2.430	86
U-238	0	2.71	2.71	~ 10	0	
Nat. U	4.18	3.40	7.58	~ 10		55
Pu-239	741.6	271.3	1012.9	8.5	2.890	73
Pu-241	1007.3	368.1	1375.4	12.0	2.934	73

Neutron Reaction Rates



Neutron Flux

Neutron flux ϕ is defined as the number of neutrons n multiplied by their velocity v

$$\phi = nv \qquad \frac{\text{number}}{\text{cm}^3} \times \frac{\text{cm}}{\text{s}} = \frac{\text{number}}{\text{cm}^2\ \text{s}}$$

Since neutron flux ϕ has units of $cm^{-2}s^{-1}$ it can be considered as the number of neutrons passing through a particular cross sectional area per unit time

Definitions

Macroscopic cross-section

(Cross-section density in material)

$\Sigma = N\sigma$ $\left(\frac{1}{cm}\right)$ or $\left(cm^{-1}\right)$

N = Nuclei per unit volume $\left(\frac{\text{nuclei}}{cm^3}\right)$

σ = Microscopic cross-section $\left(cm^2\right)$

Neutron flux

(Neutrons passing through given area per second)

$\phi = nv$ $\left(\frac{\text{neutrons}}{cm^2\,s}\right)$

n = Neutrons per unit volume $\left(\frac{\text{neutrons}}{cm^3}\right)$

v = Neutron velocity $\left(\frac{cm}{s}\right)$

Reaction rate

(Reaction rate of neutrons with material)

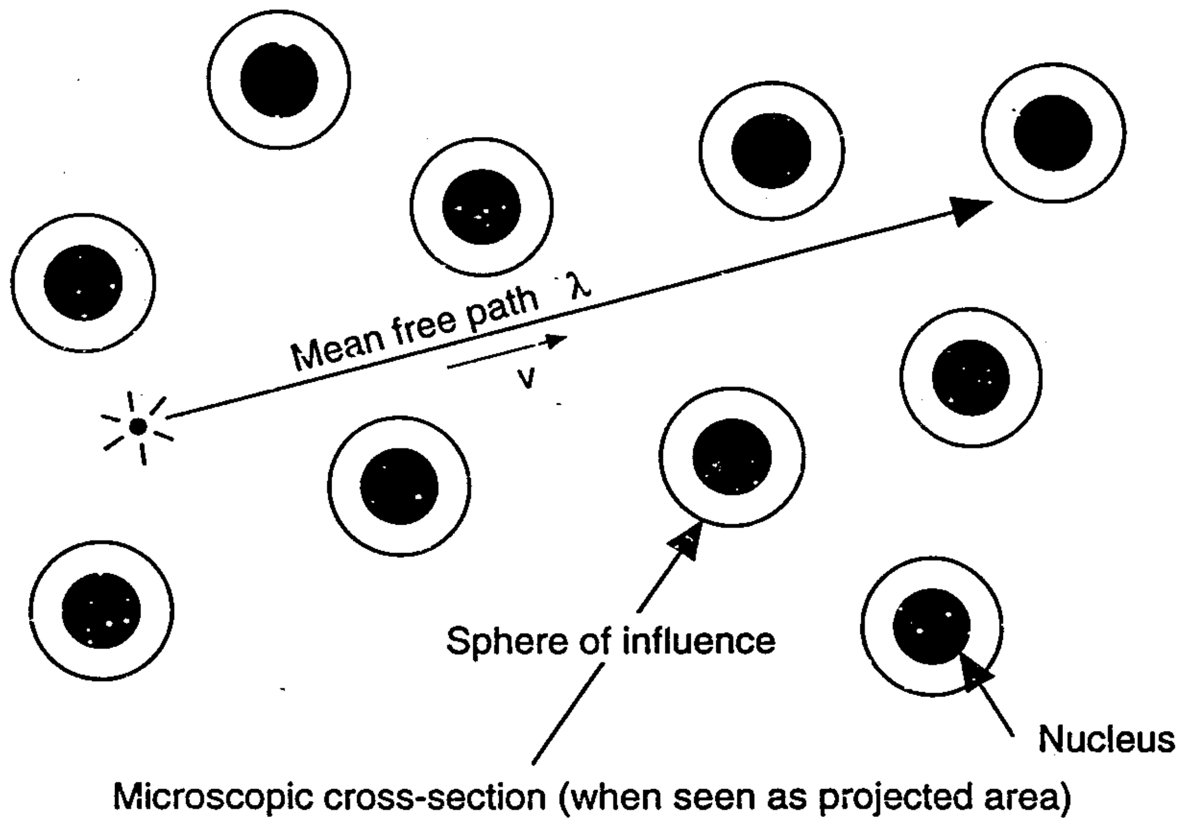
$R = \phi\Sigma$ $\left(\frac{\text{reactions}}{cm^3\,s}\right)$

ϕ = Neutron flux $\left(\frac{\text{neutrons}}{cm^2\,s}\right)$

Σ = Macroscopic cross-section $\left(\frac{1}{cm}\right)$

Neutron Mean Free Path

Macroscopic cross-section gives density of microscopic cross-sections of atoms in material



$$R = \Sigma\phi = \Sigma nv \text{ -------- } (1)$$

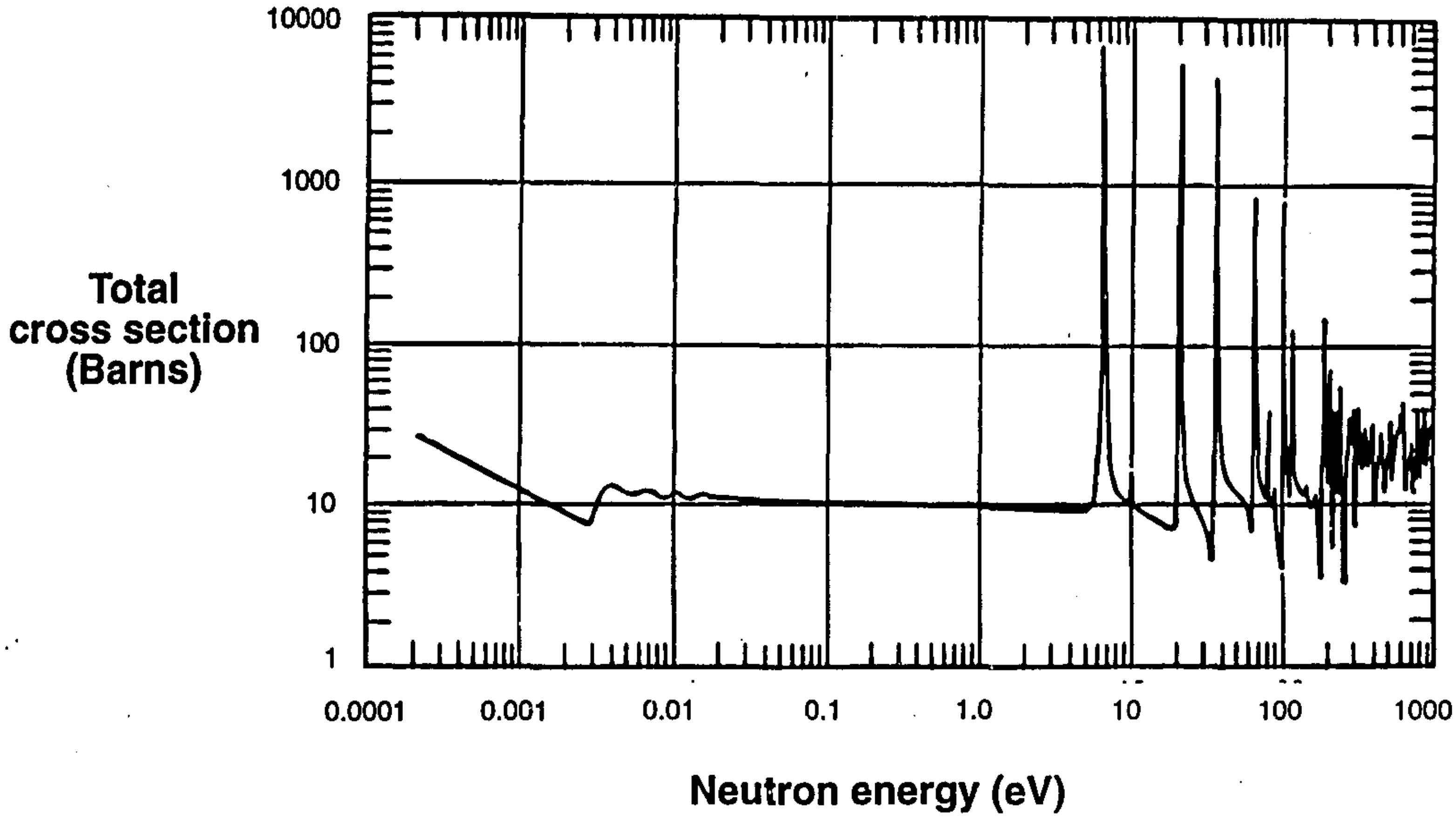
But R = Number of neutrons $\times \dfrac{\text{Distance}}{\text{Second}} \times \dfrac{1}{\text{Mean distance}}$

$$R = \frac{nv}{\lambda} \text{ -------------- } (2)$$

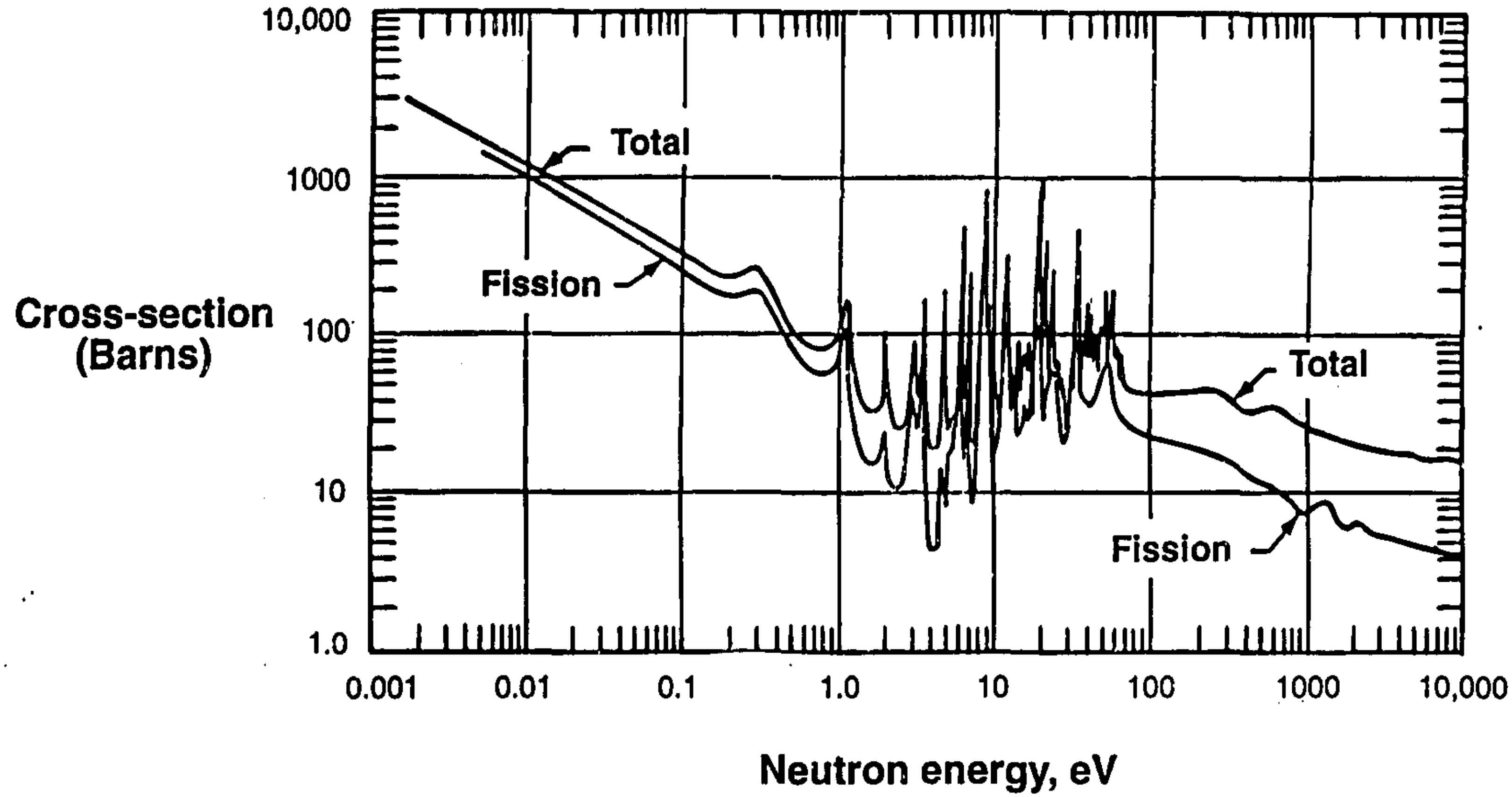
From (1) And (2)

$$\lambda = \frac{1}{\Sigma} \text{ cm}$$

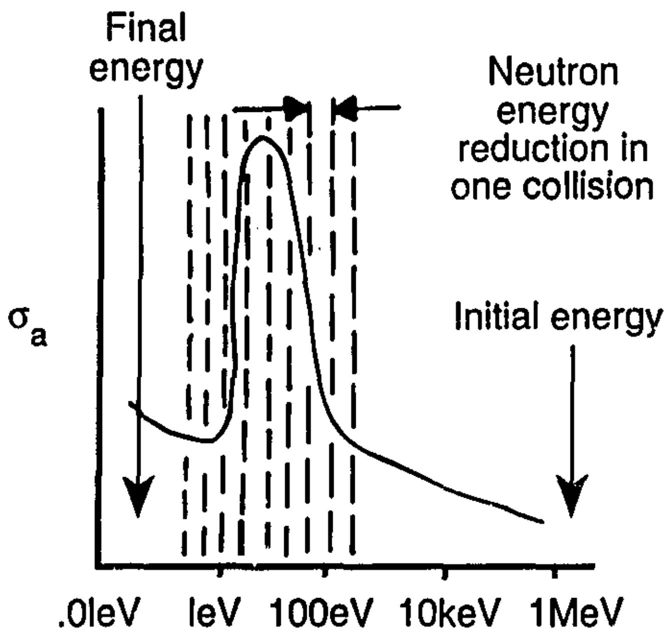
Total Cross-Section of U-238



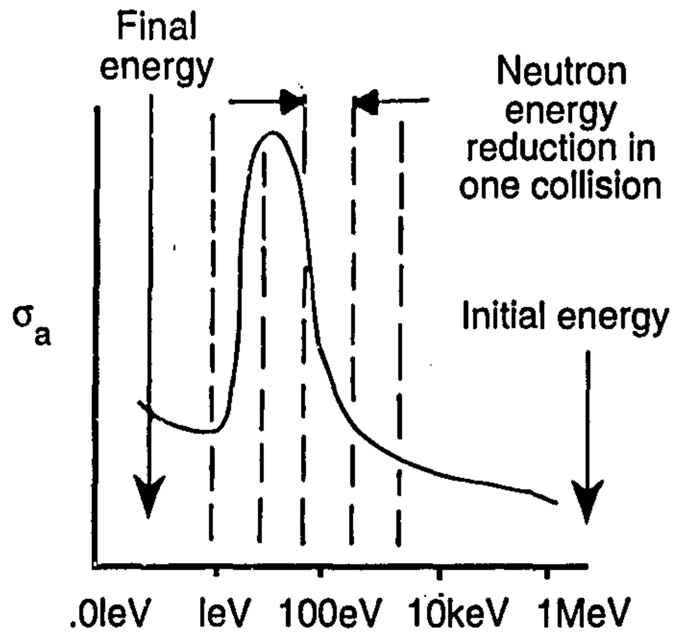
Cross-Sections of U-235



The Fraction of the Neutron's Energy that is Lost Per Collision is Small on the Left and Large on the Right



Moderator 1



Moderator 2

Change of Multiplication Factor with Moderator Isotopic (PLNGS)